AIAA 81-4129

Spin Jet Damping of Rocket-Assisted Projectiles

Charles H. Murphy*
Ballistic Research Laboratory, Aberdeen Proving Ground, Md.

Introduction

THE thrust of a rocket is produced by the high-speed discharge of gases to the rear of the rocket. In effect, the rearward linear momentum of the rocket gases is balanced by an increased forward momentum of the rocket. In the case of a rocket with pitching and yawing motion, these gases also possess angular momentum in pitch and yaw and a careful dynamics analysis shows a damping of the pitching and yawing motion associated with this gaseous angular momentum. This pitch jet damping is usually fairly small. For small rocket-assisted projectiles (RAP's), it is completely negligible.

A spinning RAP, however, ejects gases with spin angular momentum and the effect of this lost angular momentum should be considered. In this Note an expression for the spin jet damping of a rocket is derived and the size of this effect is computed for two RAP's: the 155 mm M549 and the 8-in. M650. The predicted change in spin will then be compared with actual flight measurements made by sunsondes with onboard telemetry transmitters.[1,2]

Theory

We will use the usual nonrolling aeroballistic x, $\tilde{y}$, $\tilde{z}$ axes with the x axis along the rocket's axis of symmetry and the $\tilde{z}$ axis selected to be initially downward-pointing. If the angular velocity of the rocket is $(p,\tilde{q},\tilde{r})$, the angular velocity of the coordinate system is $(0, \tilde{q},\tilde{r})$. The usual angular momentum theorem states that the derivative of the angular momentum of a constant-mass system is equal to the sum of the external moments. If the angular momentum has components $(H_x,H_{\tilde{y}},H_{\tilde{z}})$, the standard vector calculus yields

$$(\dot{H}_x,\dot{H}_{\tilde{y}},\dot{H}_{\tilde{z}}) + (0,\tilde{q},\tilde{r}) \times (H_x,H_{\tilde{y}},H_{\tilde{z}}) = (M_x,M_{\tilde{y}},M_{\tilde{z}}) \tag{1}$$

We will assume that the x axis of the rocket is a normal axis of inertia and the transverse moments of inertia, I_t, are equal. We will further assume that the rocket grain burns symmetrically so that the mass symmetry is preserved. At time t_0 the rocket has a mass m and an angular momentum

$$\vec{H} = (I_x p, I_t\tilde{q}, I_t\tilde{r}) \tag{2}$$

At time $t_0+\Delta t$ the rocket has a mass $m+\Delta m$ (Δm is negative) and a mass of gas, $-\Delta m$, has been ejected from the nozzle. In order to use Eq. (1) for a constant-mass system, we must determine the angular momentum of the rocket plus ejected gas at $t+\Delta t$ and from that compute $(\dot{H}_x,\dot{H}_{\tilde{y}},\dot{H}_{\tilde{z}})$.

The ejected gas has the shape of a disk with the radius R_n of the rocket nozzle and a thickness $u_J\Delta t$. Its angular momentum by definition is

$$h_J = \rho_J \iiint (\vec{R}_J \times \vec{V}_J) R\,\mathrm{d}R\,\mathrm{d}\theta\,\mathrm{d}x \tag{3}$$

where ρ_J is the average jet density, and $\vec{R}_J$ and $\vec{V}_J$ are the position and velocity vectors of the jet gas in this disk:

$$\vec{R}_J = (x, R\cos\theta, R\sin\theta) \qquad \vec{V}_J = (u_J, \tilde{v}_J, \tilde{w}_J) \tag{4}$$

Received July 30, 1980. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Chief, Launch and Flight Division. Associate Fellow AIAA.

The transverse motion of the exhaust gases is assumed to be the sum of the pitching motion of the nozzle and a rotationally symmetric circumferential motion due to projectile spin:

$$\tilde{v}_J = \tilde{r}X_n - pR_n f(\hat{R})\sin\theta \tag{5a}$$

$$\tilde{w}_J = -\tilde{q}X_n + pR_n f(\hat{R})\cos\theta \tag{5b}$$

where X_n is the distance from the rocket's center of mass to the nozzle exit and where

$$\hat{R} = R/R_n \qquad f(0)=0 \qquad f(1)=1$$

Equations (5) allow the angular momentum of the gas emitted in the time interval Δt to be written in a very simple form:

$$\vec{h}_J = (R_n^2 Fp, X_n^2\tilde{q}, X_n^2\tilde{r})(-\Delta m) \tag{6}$$

where

$$F = 2\int_0^1 \hat{R}^2 f(\hat{R})\,\mathrm{d}\hat{R}$$

Equation (6) can be used to obtain the angular momentum of the system at time $t+\Delta t$:

$$H_x + \Delta H_x = (I_x + \Delta I_x)(p+\Delta p) - (R_n^2 F\Delta m)p \tag{7a}$$

$$H_{\tilde{y}} + \Delta H_{\tilde{y}} = (I_t + \Delta I_t)(\tilde{q}+\Delta\tilde{q}) - (X_n^2\Delta m)\tilde{q} \tag{7b}$$

$$H_{\tilde{z}} + \Delta H_{\tilde{z}} = (I_t + \Delta I_t)(\tilde{r}+\Delta\tilde{r}) - (X_n^2\Delta m)\tilde{r} \tag{7c}$$

Equations (7) can be simplified by Eq. (2), divided by Δt, and the limit taken as $\Delta t \to 0$ to yield the derivatives of the angular momentum components. Then Eq. (1) becomes

$$I_x\dot{p} = J_p p + M_x \tag{8}$$

$$I_t(\dot{\tilde{q}} + i\dot{\tilde{r}}) - iI_x p(\tilde{q}+i\tilde{r}) = J_q(\tilde{q}+i\tilde{r}) + (M_{\tilde{y}} + iM_{\tilde{z}}) \tag{9}$$

where

$$J_p = \dot{m}R_n^2 F - \dot{I}_x \qquad J_q = \dot{m}X_n^2 - \dot{I}_t$$

Discussion

J_q is the derivative of the change in pitch moment of inertia of the propellant gases and is negative. Thus, this term damps the pitching motion, producing the well-known pitch jet damping effect. As we shall see, for small RAP's the pitch jet damping moment is much smaller than the aerodynamic damping moment and has no measurable effect on the pitching motion.

If we assume that the radial variation in circumferential velocity is linear like that of a spinning rigid body, then $f(\hat{R}) = \hat{R}$ and $F = ½$. For this case, J_p is the derivative of the change in spin moment of inertia of the propellant gases. Since J_p is positive, this term represents a spin jet *un*damping. Since it is unreasonable to expect the spin angular momentum of the emitted gases to drop to the low value associated with $f(\hat{R}) = \hat{R}$, this value yields a maximum value of J_p and we would expect the actual value to be less:

$$J_p = \epsilon(J_p)_{\max} \qquad (0 \le \epsilon \le 1) \tag{10}$$

where

$$(J_p)_{\max} = \dot{m}(R_n^2/2) - \dot{I}_x$$

The aerodynamic moment contains a spin damping term in p and a pitch damping term in $\tilde{q}+i\tilde{r}$. In order to compare the aerodynamic terms with the jet damping terms, we will nondimensionalize the jet damping moments in the same way

Table 1 Parameters for two rocket-assisted projectiles

	155 mm M549	8-in. M650
Burn-time, s	2.68	2.95
$\dot{m}$, kg/s	−1.12	−1.86
$\dot{I}_x$, kg-m²/s	−0.00285	−0.00714
$\dot{I}_t$, kg-m²/s	−0.0379	−0.0803
R_n, m	0.0222	0.0317
X_n, m	0.326	0.365
V, m/s	481	603
C_{l_p}	−0.011	−0.011
$C_{M_q}+C_{M_{\dot\alpha}}$	−13.2	−11.9
$(C_{J_p})_{max}$	0.019	0.012
C_{J_q}	−0.61	−0.34

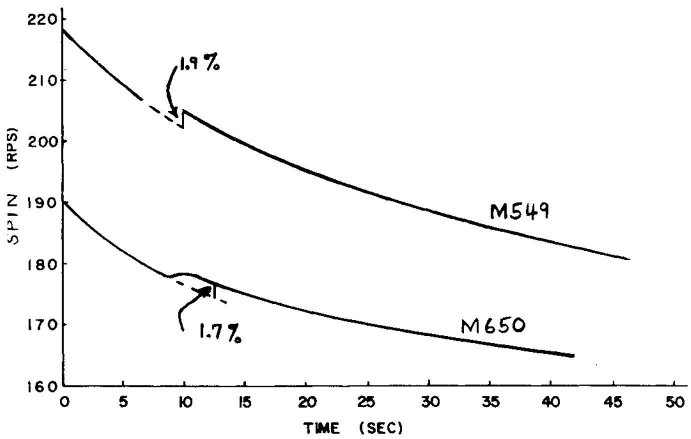

Fig. 1 Measured spin histories for two rocket-assisted projectiles showing the percentage spinup during burning.

that the aerodynamic moments are nondimensionalized[3]:

$$J_p = \tfrac{1}{2}\rho VSl^2 C_{J_p} \qquad J_q = \tfrac{1}{2}\rho VSl^2 C_{J_q} \tag{11}$$

In Table 1, the average values of $\dot{I}_x, \dot{I}_t, \dot{m}$ and other appropriate parameters are given for two rocket-assisted projectiles, the 155 mm M549 and the 8-in. M650. According to this table, the pitch jet damping coefficient is much smaller than the aerodynamic pitch damping coefficient and can be neglected, but the spin jet damping coefficient is the same size as the aerodynamic spin damping coefficient. We would therefore expect the spin to be affected by this term during burning.

Another way to estimate the effect of the spin jet damping is to consider Eq. (8) for no aerodynamic moment:

$$\dot{p}/p = -\epsilon\gamma(\dot{I}_x/I_x) \tag{12}$$

where

$$\gamma = -(J_p)_{max}/\dot{I}_x$$

γ can be computed from the table and is 0.90 and 0.87 for the M549 and M650, respectively. Thus the percentage change in spin during burning can be approximately related to the percentage change in the spin moment of inertia:

$$\frac{\Delta p}{p} = -\epsilon\gamma\left(\frac{\Delta I_x}{I_x}\right) = \begin{cases} 0.046\epsilon & \text{(M549)} \\ 0.033\epsilon & \text{(M650)} \end{cases} \tag{13}$$

Experimental Results

Recently yawsonde data have been obtained[4,5] for the spin during burning of both the M549 and the M650. Sample spin histories for each RAP are given in Fig. 1. The spin curve before burning was extended through burning and the percentage change in spin during burning was determined. It was 1.9% for the M549 and 1.7% for the M650. These values correspond to ϵ values of 0.41 and 0.52, respectively.

Conclusions

1) A theoretical model for spin jet damping has been derived which predicts a percentage increase in spin proportional to the percentage decrease in the spin moment of inertia.

2) Experimental results show approximately 2% increase in spin, which is about half the predicted maximum increase.

References

[1] Mermagen, W.H., "Measurements of the Dynamic Behavior of Projectiles over Long Flight Paths," *Journal of Spacecraft and Rockets*, Vol. 8, April 1971, pp. 380-385; see also Ballistic Research Laboratories Memorandum Rept. No. 2079, Nov. 1970.

[2] Murphy, C.H., "Effect of Large High-Frequency Angular Motion of a Shell on the Analysis of Its Yawsonde Records," Ballistic Research Laboratories Memorandum Rept. No. 2581, Feb. 1976.

[3] Murphy, C.H., "Free Flight Motion of Symmetric Missiles," Ballistic Research Laboratories Rept. No. 1216, July 1963.

[4] Platou, A.S., "Yawsonde Flights of 155mm Non-Conical Boattail Projectiles and the 155mm M549 Projectile at Tonopah Test Range—October 1977," USA ARRADCOM Ballistic Research Laboratory Memorandum Rept. No. ARBRL-MR-02881, Nov. 1978.

[5] Oskay, V., Clay, W.H., and Klawa, M., "8-Inch PXR 6263 (Temperature Instrumented M650 RAP) Tests at Yuma Proving Ground," USA ARRADCOM Ballistic Research Laboratory Memorandum Rept. No. ARBRL-MR-03037, Aug. 1980.

AIAA 81-4130

Inversion of a Class of Complex Matrices

David R. Glandorf*
Lockheed Engineering and Management Services Company, Houston, Texas

Introduction

INVERSION of complex matrices is required for the solution of many problems. One approach to the inversion problem is simply to use complex arithmetic with a standard inversion algorithm. Another approach is to use various identities to transform the problem into one which requires only real arithmetic. The latter approach is usually not as efficient as the first with respect to either speed or storage requirements; and sometimes requires certain nonsingularities which may not be guaranteed a priori.[1-4]

This Note is concerned with the inversion of a certain class of complex matrices which frequently arise in practice, namely, nonsingular matrices for which each column (or row) is either real or the complex conjugate of another column (or row). Probably the most common application is in the diagonalization of a real matrix A by a similarity transformation $T^{-1}AT$ where T is the modal matrix constructed from the right eigenvectors of A. The inverse of T is the transpose of the matrix of left eigenvectors of A, so the similarity transformation also provides the left eigenvectors as a byproduct.

Received Aug. 25, 1980; revision received Nov. 7, 1980.

*Advanced Systems Engineering Specialist.